in cross section, where $2a$ is now the dimension of either side of the square.

In the dominant mode the electric field with corner excitation is

$$E_z = E_0 \left[\sin \frac{\pi}{4a}(x+y) \cos \frac{\pi}{4a}(x-y)\right] \quad (9)$$

for which the resonance is at $ka = \pi/2$ and the mode-geometry parameter

$$R/Q = \eta \frac{b}{a} \frac{1}{ka}. \quad (10)$$

Squares of side $2a = 1.28$ in, which resonate at the same frequency as the circular disk, were fabricated. Using the same procedure as before, the average Q was found to be 163 compared with an average directly measured value of 177. Based on the value of 163, the Q due to radiation loss was found to be 352; based on the value of 177, the Q due to radiation loss was 425.

The electric field of the second square-plate mode is

$$E_z = E_0 \sin \frac{\pi}{2} \frac{x}{a} \sin \frac{\pi}{2} \frac{y}{a} \quad (11)$$

and resonates at

$$ka = \frac{\pi}{\sqrt{2}}.$$

The R/Q parameter in this case is again

$$\frac{R}{Q} = \eta \frac{b}{a} \frac{1}{ka} \quad (12)$$

which allows the comparisons made before. These results as well as the preceding square-plate results are similarily tabulated in the second portion of Table I. The previous comments regarding accuracy apply here also.

In the preceding examples, the Q's are high and the resonant frequencies widely separated so that there is considerable assurance that only the desired mode was excited. The correspondence between the measured and the calculated resonant frequencies should be noted.

Prior to performing the experimental work there was some concern regarding the possible influence of reflections from surrounding objects. Fortunately, these effects were found to be negligible as long as any perturbing objects were an inch or more from the edge of the resonator. This insensitivity is probably due to the relatively small thickness of substrate-to-wavelength ratio.

Summary

An additional route to the evaluation of radiation from planar-circuit cavity resonators is gained from a calculation of the R/Q parameter which, in effect, allows a Q measurement to be replaced by a resistance measurement. Preliminary results for two simple planar resonators are given.

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Measurement of Microwave Loss Tangent by Means of Microwave Resonator Bridge

IVAN KNEPPO AND MICHAL WEIS

***Abstract*—A new cavity method of measurement of microwave loss tangent of a low-loss material is described. The method consists of the comparison of the reflection coefficient of the resonator filled with dielectric with the reflection coefficient of the reference resonator by means of microwave resonator bridge. Basic theoretical relations of a simplified resonator bridge are derived, a fundamental scheme of the measuring set is given, and some results of verified measurements presented. The obtained results confirm the suitability of the proposed method, especially for measuring low-loss dielectric materials, since the principle of comparison renders the method highly sensitive to small parameter differences of the resonator brought about by the measured sample.**

Nomenclature List

a_1	Equivalent voltage of the wave incident in port 1.
b_i	Equivalent voltage of the wave scattered from port i.
C	Capacitor of the equivalent series resonant circuit.
i	Integer denoting port of hybrid junction.
j	Square root of -1.
L	Inductor of the equivalent series resonant circuit.
P_1	Microwave power fed into the MRB in the unperturbed condition.
P_1'	Microwave power fed into the MRB in the perturbed condition.
P_4	Output power of the MRB in the unperturbed condition.
P_4'	Output power of the MRB in the perturbed condition.
R	Factor Z_{03}/Q_{03}.
Z_{02}, Z_{03}	Normalized geometrical factor of the measured or reference resonator, respectively.
Z_c	Characteristic impedance of the line.
Γ_2, Γ_3	Reflection coefficient of the measured or reference resonator, respectively.
Γ_4	Reflection coefficient of the detector.
Γ_{in}	Reflection coefficient of the MRB in the unperturbed condition.
Γ_{in}'	Reflection coefficient of the MRB in the perturbed condition.
δ	Angle of dielectric losses.
ε	Relative permitivity.
Θ	Phase shift.
θ	Relative difference of the resonant frequencies.
κ	Factor Q_{03}/Q_{02}.
v	Relative detuning of the reference resonator.
χ	Factor defined by the expression (A9).
ω_{02}, ω_{03}	Resonance frequency of the measured or reference resonator, respectively.

I. Introduction

The cavity methods, especially the cavity perturbation methods, are often used to measure microwave dielectric losses, [1], [2], but in measuring materials of small dielectric losses it is very difficult to determine the change of the cavity quality factor by only the difference of the half-power bandwidth of the resonance curve, and the obtained results are thus unsatisfactory. The authors present a new and highly sensitive method for measuring

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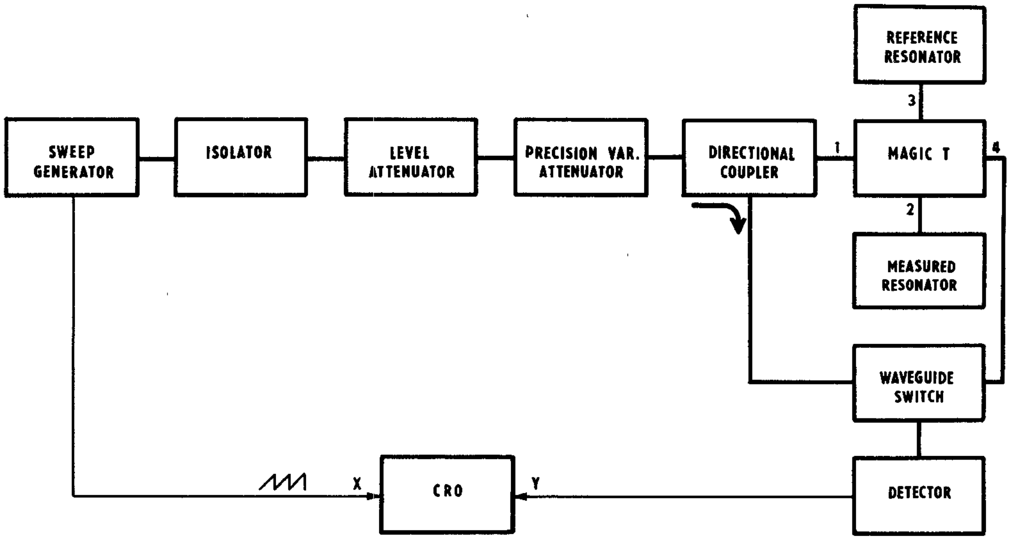


Fig. 1. Experimental arrangement for loss tangent measurement by MRB method.

dielectric losses of materials by which the difference of the quality factor is not calculated from changes of the bandwidth but from the difference of the output signal of the microwave resonator bridge (MRB). The MRB consists of the two resonators connected at the two ends of the througharms of the magic T. One resonator is that whose parameters are to be measured (with or without sample), and the second is the reference resonator. The signal of the microwave generator is fed into the H-plane arm and the matched detector is connected to the E-plane arm of the magic T. The described method is, in principle, a method of comparison because, in the hybrid T, the reflection coefficients of both resonators are compared and the detector indicates the difference signal. As with other comparison methods, here, too, it is necessary to know the parameters of the reference standard resonator, and the method is more convenient for measuring the relative differences of resonator parameters than for determining their absolute value. The MRB correctly allows one to indicate very small differences of the quality factor of the measured resonator, brought about by the applied sample.

II. Basic Relations

The resultant relations for calculating the relative quality factor difference of the measured resonator (before and after sample insertion) are obtained by solving (A8) with regard to the unknown quantity χ, and from (A9), i.e.,

$$\chi = \kappa^{-1} \frac{(\kappa - 1)(|b_4'|/|b_4|)_0 + 1 + \kappa R}{1 + \kappa R - R(|b_4'|/|b_4|)_0} \tag{1}$$

$$\Delta Q_2/Q_{02} = \chi^{-1}(1 - \chi + \Delta Z_2/Z_{02}). \tag{2}$$

Expression $\Delta Z_2/Z_{02}$, appearing on the right side of (2), may be determined on the basis of measuring the frequency shift $\Delta\omega_0$ of the resonator after inserting the measured sample into the resonator. Because the dielectric sample exerts an influence only upon the electric-field component in the resonator, the relative frequency shift is $\Delta\omega_{02}/\omega_{02} = -0.5(\Delta C/C)$ and the relative difference of the geometrical factor is $\Delta Z_2/Z_{02} = -0.5(\Delta C/C)$. From this results follows straightforward $\Delta Z_2/Z_{02} = \Delta\omega_{02}/\omega_{02}$. Here we must note that in all our considerations the influences of external circuits on resonator losses were neglected and, therefore, in the derived relations only the unloaded quality factors of the resonators are present. In the real case, the transmission-line losses are transmitted by means of a coupling circuit into the resonator, thus decreasing its resulting loaded quality factor. Using an analogous approach, as in the resonator-bridge analysis, we can show that the derived relations remain valid. In this case, the loaded quality factors Q_{Li}, where $i = 2, 3$, instead of the unloaded quality factors Q_{0i}, where $i = 2, 3$, must be used. However, the influence of the sample on the coupling circuit of resonator is neglected.

III. Measurement

Fig. 1 shows the arrangement of the system for measuring the dielectric losses of materials in X band by the described method. Measured and reference resonator, both of equal shape, with equal modes and equal coupling circuits are connected to the ends 2 and 3 of the througharm of the magic T. The stabilized microwave power of the sweep generator is fed into the H-plane arm where the level attenuator, the precise calibrated variable attenuator, and directional coupler are also connected. The matched detector may be alternately connected to the E-plane arm of the magic T or in the forward-direction auxiliary arm of the directional coupler by means of the waveguide switch. In the first position, the detector indicates the output signal of the MRB and its changes during the measurements. In the second position, the detector monitors the power fed into the MRB from the microwave generator. Since for the measurement of the relative proportion of power the precise attenuator is used and the detector serves only as a level indicator, no special claims are laid on the selection of the detector and its characteristics.

The output power of the MRB without sample is $P_4 = 0.5(|b_4|^2/Z_c)$, with sample $P_4' = 0.5(|b_4'|^2/Z_c)$, if a matched detector was presumed. Microwave power entering into the MRB is $P_1 = 0.5(|a_1|^2/Z_c)(1 - |\Gamma_{in}|^2)$ without sample and $P_1' = 0.5(|a_1|^2/Z_c)(1 - |\Gamma_{in}'|^2)$ with sample, where Γ_{in}, Γ_{in}' are reflection coefficients of MRB without or with sample, respectively. To be able to use (1) and (2) from the previous section, it is necessary to preserve a constant level of input power of the MRB without or with sample, i.e., $P_1 = P_1'$, by means of the level attenuator. Then we may write $|b_4|/|a_1| = (P_4/P_1)^{1/2}$ and $|b_4'|/|b_4| = (P_4'/P_4)^{1/2}$, and ratios of equivalent voltages in the relations in Section II may be determined by measuring the power ratios in the corresponding arms of the measurement set.

Using a microwave sweep generator that sweeps frequency in the vicinity of the resonance frequencies of the resonators and by displaying the detected MRB output signal on the CRT, the process of MRB balancing may be easily carried out.

The entire process of measurement consists of the calibration of the measurement set (factor R determination) and of the measurement proper.

Calibration: 1) The detector is connected in the E-plane arm of the magic T. The tuning of the empty measured resonator to the resonance frequency of the reference resonator, i.e., so the displayed peak is minimal. Check the peak height. 2) Switch the detector by waveguide switch into the auxiliary arm of the directional coupler and set the precision variable attenuator in a way to make the signal reach the peak level of the preceding step. 3) Check the difference values of the precision attenuator setting. Calculating the square root of ratio $(P_4/P_1)^{1/2}$ and calculating factor R in terms of (A11). With this step the calibration is concluded.

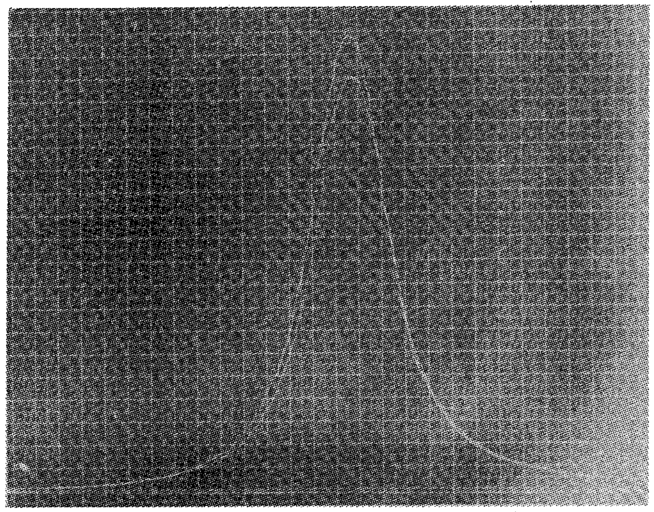
Measurement: 4) Insert the sample into the measured resonator. 5) Retune the measured resonator to the original resonance frequency, i.e., so the displayed peak is minimal again. Check the resonance frequency shift $\Delta\omega_0$. 6) Switch the detector into the auxiliary arm of the directional coupler and check the level of the MRB input power. Possible differences are balanced by the level attenuator. 7) Repeat switching the detector into the output of the MRB and set the precision variable attenuator so that the peak height of the output signal is the same level as it was with the empty resonator. Read the difference of the precision attenuator and calculate $(P_4'/P_4)^{1/2}$. 8) Calculate the relative quality-factor change of the measured resonator with the aid of (1) and (2). The calculated values of the frequency shift and quality-factor changes may be used in the known manner to determine the complex permittivity of the tested material [4].

IV. Discussion

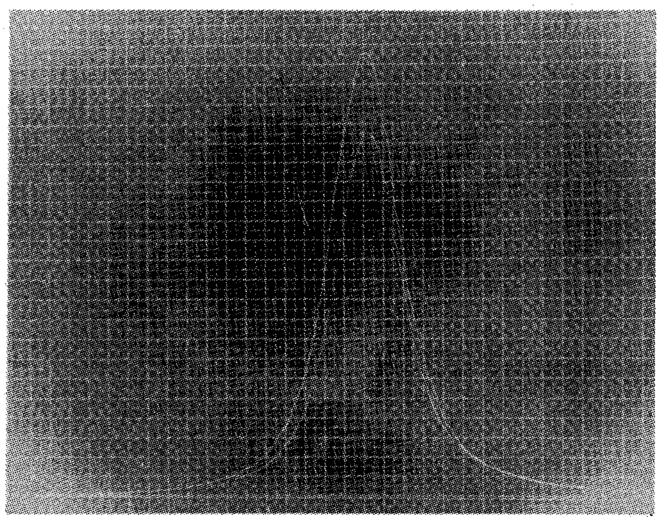
The described MRB method of material loss tangent measurement has been verified by a series of experimental samples of Teflon and organic glass. Fig. 2(a), (b) illustrate the records of output signals of the MRB without and with samples of the materials referred to. For comparison Fig. 3(a), (b) show the records of the resonance curves of the measured resonator without sample and with the same samples as in the preceding case. It can be seen that especially the sample of low-loss material (Teflon) inflicts changes on the resonance curve that are practically not discernible from the resonance curve of the empty resonator, where as output signal of the MRB indicates distinct differences. MRB responds sensitively to changes of quality factors of the connected resonators. On the other hand, it indicates with special distinctness differences in their resonance frequencies. A difference of 1 MHz in X band may be reliably registered. This advantage, however, may be the cause of inaccurate measurement of material losses. Restricted care in tuning the resonator may create a situation wherein the height of the set peak of the output signal does not comply with the correct minimal peak. It is thus necessary to have the tuning mechanism of the applied cavity resonator provide for smooth, reproducible, and fluent tuning and to have it manufactured accurately and without mechanical backlash.

V. Conclusions

An MRB method of measuring microwave losses of material, especially dielectrics with low losses, has been presented. In addition to high sensitivity, the described method has the following advantages. 1) The accuracy of the measurement is not affected by parasitic FM of the microwave signal. 2) By using a bridge network, temperature effects on the properties of the measured resonator are inherently compensated for, if the reference resonator is made of the same material as the measured one. 3) The measurement network is simple and no expensive measuring devices are required, such as a digital frequency meter or a stable and accurate microwave generator, which are instrumental in measuring the width of the resonance curve. 4) The signal indicating the parameter changes of the resonator appears in the form of electric voltage. This form is most suitable for a further use of the signal: recording, processing, and control (if the microwave network forms part of an automatic system). The output signal of the detector may be amplified, thus enhancing the sensitivity of the indication of parameter changes. 5) The measurement of the performance ratios of the microwave signal by means of a precision attenuator eliminates the need for accurate calibration of the applied detector. The detector is used only to indicate the constant level of the microwave output at the checkpoints of the measurement network. 6) The MRB measurement network may be completed by a feedback loop of

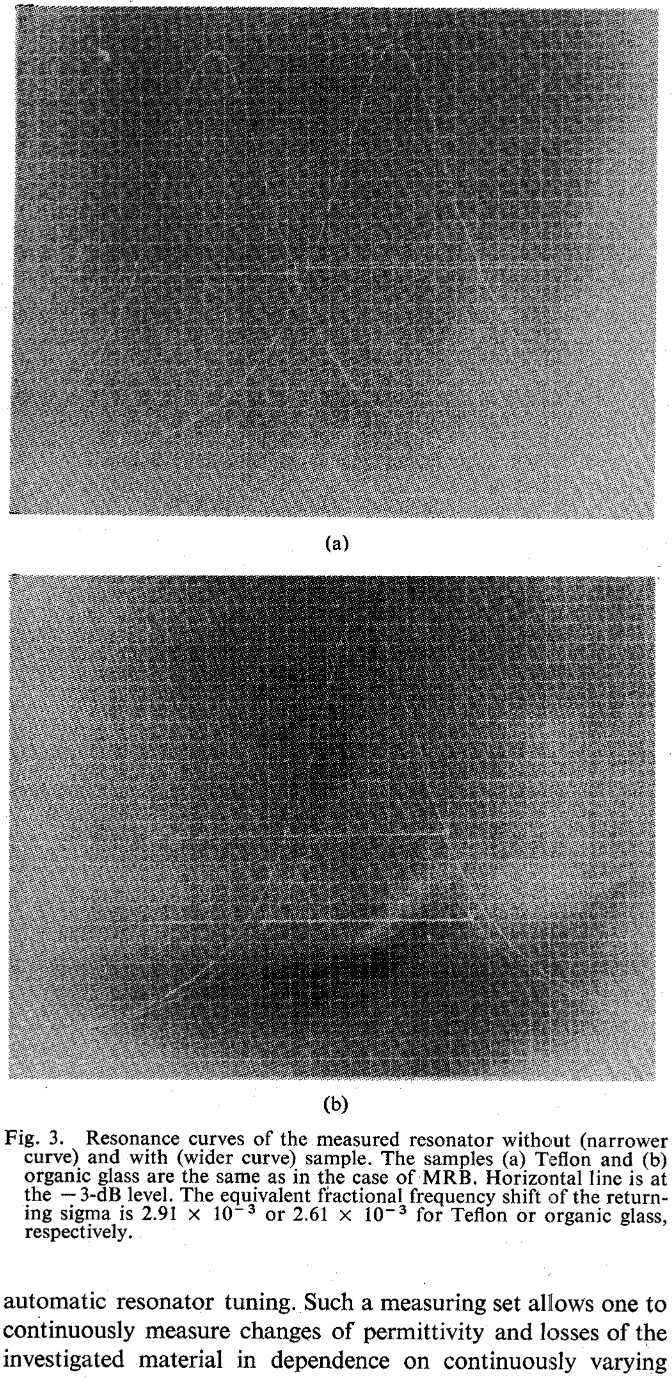


(a)



(b)

Fig. 2. Typical records of the output signal of the MRB without (lower curve) and with (higher curve) sample. (a) Teflon rod 6-mm diameter, 22-mm length, $\varepsilon = 2.1$, $tg\delta = 2 \times 10^{-4}$. (b) Organic glass rod 6-mm diameter, 10-mm length, $\varepsilon = 2.5$, $tg\delta = 43 \times 10^{-4}$.



(a)

(b)

Fig. 3. Resonance curves of the measured resonator without (narrower curve) and with (wider curve) sample. The samples (a) Teflon and (b) organic glass are the same as in the case of MRB. Horizontal line is at the −3-dB level. The equivalent fractional frequency shift of the returning sigma is 2.91×10^{-3} or 2.61×10^{-3} for Teflon or organic glass, respectively.

automatic resonator tuning. Such a measuring set allows one to continuously measure changes of permittivity and losses of the investigated material in dependence on continuously varying physical parameters of the environment: of temperature, illumination, pressure, moisture, etc. [7]. The errors caused by the mismatch of the detector are negligible, and the other failures brought about by the parameter dispersion of applied components may be eliminated by calibrating the network prior to measurement. The disadvantages of the suggested MRB method are that the network requires the application of two equal microwave resonators and the knowledge of the loaded quality factors. The magic T used has to be electrically in complete symmetry within the working frequency range.

Appendix I
Theory of MRB

Consider a symmetric, lossless, and matched hybrid junction, with two resonators connected at the two ends 2 and 3 of the through arm, as shown in Fig. 4. Consider further a reflectionless

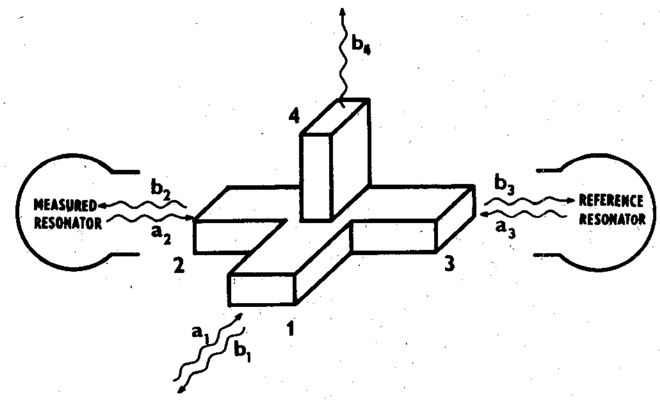


Fig. 4. Microwave resonator bridge.

termination of arm 4 and the lengths of arms 1 and 4 chosen in such a way that the elements of the scattering matrix of the magic T be real. The equivalent voltages of the incident and reflected waves (a_1 and b_i, for $i = 1, 2, 3, 4$) in the individual ports may hence be reaily expressed by an equation in matrix form as follows:

$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & \Gamma_2 & \Gamma_3 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & \Gamma_2 & -\Gamma_3 & 0 \end{bmatrix} \begin{bmatrix} a_1 \\ b_2 \\ b_3 \\ 0 \end{bmatrix} \tag{A1}$$

where $\Gamma_2 = a_2/b_2$; $\Gamma_3 = a_3/b_3$ are reflections of the measured and reference resonator, respectively. The equivalent voltage of the incident wave falling into arm 4 is zero, on grounds of reflectionless termination. By solving the system of linear equations (A1), we give the expression for the ratio of equivalent voltages of the transmitted and incident wave in arms 4 and 1, i.e., $b_4/a_1 = 0.5(\Gamma_2 - \Gamma_3)$, or the ratio of the absolute values of these equivalent voltages

$$|b_4|/|a_1| = 0.5|\Gamma_2 - \Gamma_3|. \tag{A2}$$

The reflection coefficients of the high Q resonators in the vicinity of resonance are

$$\Gamma_2 = \frac{Z_{02}/Q_{02} - 1 + j2Z_{02}(\nu + \theta)}{Z_{02}/Q_{02} + 1 + j2Z_{02}(\nu + \theta)} \tag{A3}$$

and

$$\Gamma_3 = \frac{Z_{03}/Q_{03} - 1 + j2Z_{03}\nu}{Z_{03}/Q_{03} + 1 + j2Z_{03}\nu} \tag{A4}$$

for the measured and reference resonators, respectively. The resonators were considered with mutually close resonance frequencies and as series resonance circuits with lossless coupling. The unloaded quality factors are denoted as Q_{02} and Q_{03}, while Z_{02} and Z_{03} are normalized (measured in units of waveguide characteristic impedance Z_c) geometric factors of cavities: $Z_0 = Z_c^{-1}(L/C)^{1/2}$, if L and C are inductivity and capacity elements of the equivalent-series resonance circuit. Further, $\theta = (\omega_{03} - \omega_{02})/\omega_{03}$ is the relative difference of the resonance frequencies ω_{02} and ω_{03} of resonators and $\nu = \Delta\omega/\omega_{03}$ is the relative detuning of the reference resonator. After substituting expressions (A3) and (A4) in (A2), we get

$$|b_4|^2/|a_1|^2 = \frac{(Z_{02}/Q_{02} - Z_{03}/Q_{03})^2 + 4[Z_{02}(\nu + \theta) - Z_{03}\nu]^2}{[(Z_{02}/Q_{02} + 1)^2 + 4Z_{02}^2(\nu + \theta)][(Z_{03}/Q_{03} + 1)^2 + 4Z_{03}^3\nu^2]}. \tag{A5}$$

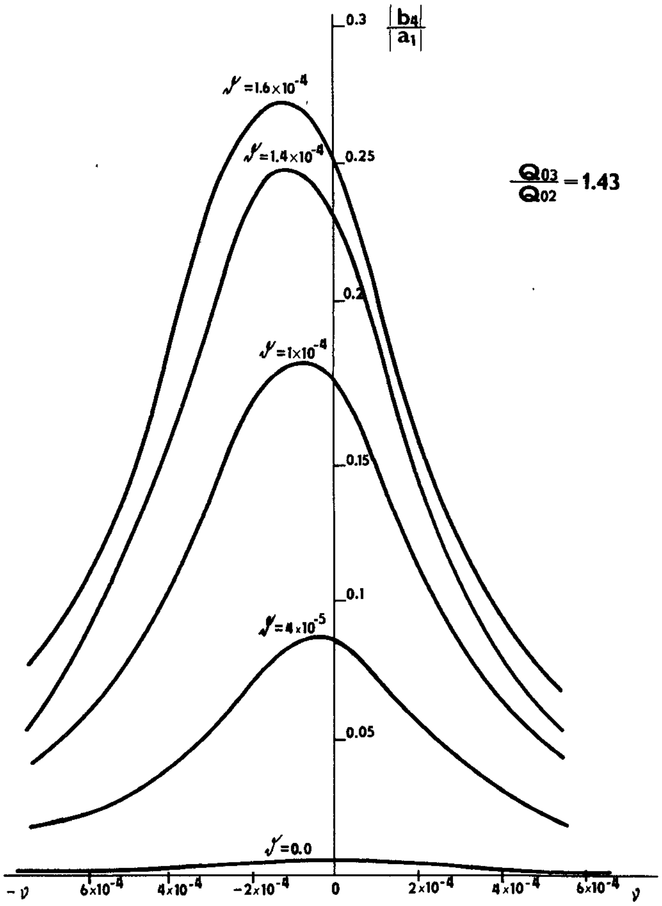


Fig. 5. Computed response of the MRB.

The response $|b_4|/|a_1|$ as a function of ν, with θ as a parameter, computed for resonators of equal shape, and with equal modes, is shown in Fig. 5. The lowest curve pertains to parameter $\theta = 0$, i.e., to the case, when both resonators are tuned to an equal resonance frequency and its maximum is

$$(|b_4|/|a_1|)_0 = \frac{Z_{03}}{Q_{03}} \frac{Q_{03}/Q_{02} - 1}{(1 + Z_{03}/Q_{02})(1 + Z_{03}/Q_{03})}. \quad \text{(A6)}$$

The small sample of a lossy dielectric put into the measured resonator inflicts a change to the geometric factor, the resonance frequency, and the quality factor of the resonator: $Z_{02} \to Z_{02} + \Delta Z_2$; $\theta \to \theta + \Delta\theta$; $Q_{02} \to Q_{02} + \Delta Q_2$. After retuning the cavity so as to make the resonance frequency achieve the original value again, the peak height of the new curve of the MRB output signal is

$$(|b_4'|/|a_1|)_0 = \frac{Z_{03}}{Q_{03}} \left[\frac{Q_{03}(Z_{03} + \Delta Z_2)}{Z_{03}(Q_{02} + \Delta Q_2)} - 1\right] \cdot \left[\left(1 + \frac{Z_{03} + \Delta Z_2}{Q_{02} + \Delta Q_2}\right)\left(1 + \frac{Z_{03}}{Q_{03}}\right)\right]^{-1} \quad \text{(A7)}$$

thus exceeding that of the MRB with an empty resonator. Provided that the absolute value of the equivalent voltage of the incident wave a_1 remains constant, the division of relation (A7) by relation (A6) yields, for the proportion of the amplitude of the transmitted wave of the MRB with sample to the transmitted-wave amplitude of the MRB without sample, the formula

$$(|b_4'|/|b_4|)_0 = \frac{(1 + \kappa R)(\kappa\chi - 1)}{(\kappa - 1)(1 + \kappa R\chi)} \quad \text{(A8)}$$

where $R = Z_{03}/Q_{03}$, $\kappa = Q_{03}/Q_{02}$, and

$$\chi = \frac{1 + \Delta Z_2/Z_{03}}{1 + \Delta Q_2/Q_{02}}. \quad \text{(A9)}$$

By using substitution expression R, κ, and χ, relation (A6) may also be readily described in a lucid form

$$(|b_4|/|a_1|)_0 = R \frac{\kappa - 1}{(1 + R)(1 + \kappa R)}. \quad \text{(A10)}$$

From obtained expressions (A10) and (A8), it follows that if the parameters Q_{03} and Z_{03} of the reference resonator are known, then the measurements of the two ratios $(|b_4|/|a_1|)_0$ and $(|b_4'|/|b_4|)_0$ allow one to determine the relative difference of the quality factor of the measured resonator, caused by the applied sample of dielectric material. As a rule, the geometric factor Z_{03} of the reference resonator is generally unknown and its measurement constitutes a difficulty [3]. It is thus more advantageous to know the unloaded quality factor Q_{02} of the empty measured resonator in advance, and the geometrical factor Z_{03} from (A10) may then be calculated

$$R = (2\kappa)^{-1}[(\kappa - 1)(|a_1|/|b_4|)_0 - \kappa - 1 \pm ([\kappa + 1 - (\kappa - 1)(|a_1|/|b_4|)_0]^2 - 4\kappa)^{1/2}]. \quad \text{(A11)}$$

If the assumption $Q_{02}' < Q_{02} < Z_{03}$ is satisfied, then we choose from the two possible solutions of (A11) the value $R < 1/\kappa$.

Appendix II
Discussion of Errors

The main sources of errors in measuring with the MRB method are the properties of real elements applied in the measurement network.

1) If the detector is not matched to the output of the MRB, its reflection coefficient is $\Gamma_4 = a_4/b_4$. The equation system (A1) will include equivalent voltage a_4 of the wave reflected from the detector, and the absolute-value ratio of equivalent voltages $|b_4|/|a_1| = 0.5|\Gamma_2 - \Gamma_3|/|1 - 0.5\Gamma_4(\Gamma_2 + \Gamma_3)|$ differs by a factor in the denominator of relation (A2). The proportion of output equivalent voltages prior to and after sample insertion is

$$(|b_4'|/|b_4|) \cdot \frac{|1 - 0.5\Gamma_4(\Gamma_2 + \Gamma_3)|}{|1 - 0.5[\Gamma_4(\Gamma_2 + \Gamma_3) + \Delta\Gamma_2\Gamma_4 + \Delta\Gamma_4(\Gamma_2 + \Delta\Gamma_2 + \Gamma_3)]|}.$$

As may be seen from the given relation, a small mismatch, if $|\Gamma_4| \ll 1$ and a small change of the reflection coefficient of the measured resonator $|\Gamma_2| \ll 1$ brings about a difference between the denominator and numerator of the second term on the right side only in small quantities of the second and third order. This is due to the fact that the effect of a small mismatch of the detector to the measurement accuracy by the MRB method is practically negligible.

2) The difference in the electric lengths of the direct arms of the magic T gives rise to a phase shift Θ between waves in arms 2 and 3. If $|\Theta| \ll 1$, then the proportion of equivalent voltages is

$$(|b_4'|/|b_4|)_0 \left[1 + \left(\frac{\Theta\Gamma_2}{\Gamma_2 + \Delta\Gamma_2 - \Gamma_3}\right)^2\right]^{1/2} \cdot \left[1 + \left(\frac{\Theta\Gamma_2}{\Gamma_2 - \Gamma_3}\right)^2\right]^{-1/2}.$$

The unequal arm length is the source of measurement error which rises with the magnitude of the reflection-coefficient change of the measured resonator. It may be removed by the accurate production of the hybrid T or by correction of measured data. The value of the correction factor may be established by measuring the output signal of the MRB, if both resonators are tuned outside the chosen working frequency band. In this case, resonators represent short circuits at the ends of the througharm of the magic T with reflection coefficients $\Gamma_2 = \Gamma_3 = -1$, and detuned $|b_4|/|a_1| = 0.5\Theta$.

3) The frequency dependence of the S-parameters of the magic T is shown as follows. In analysis of the theoretical model of the MRB it was presumed that the elements of the scattering matrix are constant and real over the entire frequency band. This presumption may not be satisfied with actual components, however, and hence the derived relations may not have accurate validity. The analytical expression of the S-parameter dependence of the hybrid junction on frequency, and hence the investigation of the influence of the frequency changes upon the accuracy of the method is not a simple matter, in general. Experimentally, two procedures are possible: a) to measure, within the selected frequency range, the parameters of the magic T and to apply them in deriving new relations analogically to the procedure explained in the theoretical section; b) to choose, on the basis of measurement in a broader frequency range, a suitable working frequency range in which the scattering parameters satisfy the presumptions of the derived theoretical model.

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Comparison Method of Measurement Q of Microwave Resonators

IVAN KNEPPO

***Abstract*—A method for the measurement of the quality factor of microwave resonators is given. It is based on the comparison of the transmitted power of the measured resonator with that of a reference resonator. The characteristics obtained by simultaneous display of the transmitted powers of the two resonators in Cartesian coordinates are analyzed. The relationship between the shape of the resulting curve and the parameters of the reference and measured resonators is discussed. The principal scheme of the X-band measuring microwave set and the measurement procedure are described. Some verification of the measurement results are presented. The method described is especially convenient for measuring small relative changes in the quality factor. It can be utilized for measuring the microwave loss tangent of materials by perturbation resonator methods.**

Nomenclature List

i	Integer denoting the arm of the microwave network.
K	Ratio of the gain increase of the vertical amplifier.
$P_i(\omega)$	Transmitted power of the ith resonator.
$P_i(\omega_{0i})$	Transmitted power of the ith resonator at resonance.
Q_{Li}	Loaded quality factor of the ith resonator.
x, y	Real variables in the Cartesian coordinates.
x_1	Point of the extreme.
δ	Difference function.
δ_1	Value of the extreme of the difference function.
θ	Relative difference of resonance frequencies.
ν	Relative change of the frequency.
ω	Frequency.
ω_{0i}	Resonance frequency of the ith resonator.

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I. Introduction

The most widely used method of measuring microwave cavity Q is the measurement of the bandwidth of the resonator. A number of bandwidth-measurement methods have been evolved based on the accurate measurement of the frequency interval between the half-power points of a resonance curve, or between the inflection points, or based on the phase shift of the modulation envelope of the transmitted signal. A detailed description of these methods is given in a number of publications; a good survey is in [1]. A common deficiency of the bandwidth methods is the difficulty in measuring small changes in the cavity Q factor which occur, for example, in perturbation methods for the measurement of the loss tangent of low-loss dielectrics. It is also a significant fact that such a measuring set, capable of detecting small changes in the cavity Q factor, is rather sophisticated, and microwave measuring instruments belonging to the highest precision class are expensive. Especially high demands are laid upon the frequency stability of the microwave generator and upon the precision of the frequency meter which is usually a digital one.

Less widespread methods, in microwave practice, bearing upon the measurement of the cavity Q factor are those based on the comparison of the measured resonator with a reference resonator, or with a reference resonance circuit with known parameters. These methods are convenient in those cases when accuracy in measuring the absolute value of the Q factor is secondary and the primary requirement is to indicate and measure the small relative change in the resonator parameters. Papers [2] and [3] present a description of a method by which the resonance curve of the measured microwave resonator is compared with that of a calibrated lower frequency resonance circuit of variable Q. Paper [4] discusses the comparison of the measured quality factor to that of the reference resonator by means of the microwave resonator bridge.

In the present short paper, a new comparison method for measuring the microwave resonator quality factor is suggested. The proposed method is based on the comparison of the powers transmitted through the measured and reference resonators. The transmitted powers of both resonators are displayed, during the sweeping, on the oscilloscope screen in Cartesian coordinates with the transmitted power of the reference resonator recorded on the x axis while the difference of the transmitted powers of the measured and reference resonators is recorded on the y axis. The shape of the curve displayed depends on the difference in the parameters of both resonators, and expressively varies with small changes in the parameters. The tuning of the measured resonator to the resonance frequency of the reference resonator is indicated very markedly. Also it is possible to read from the graph of the resultant curve all the data which are necessary for the calculation of the cavity quality factor. The proposed method is very convenient for the measurement of small changes in the quality factor and resonant frequency of the microwave resonator.